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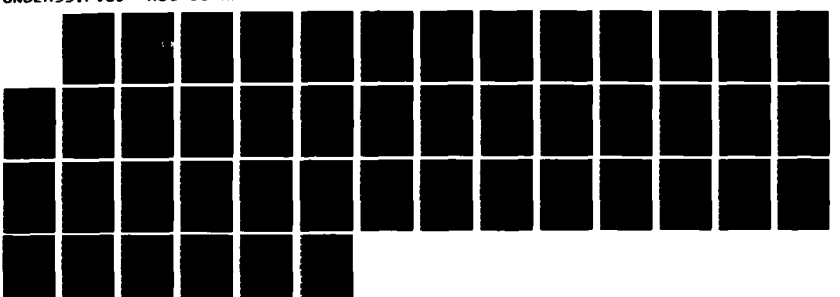

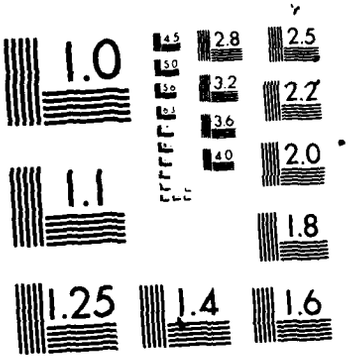

AN ENDOCHRONIC RATE-SENSITIVE CONSTITUTIVE EQUATION FOR METALS. APPLICATION TO GENERALIZED CREEP

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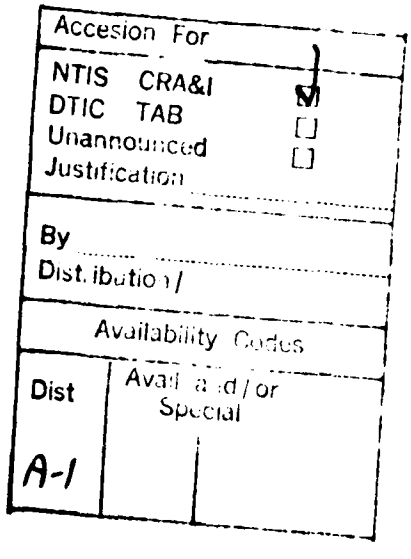

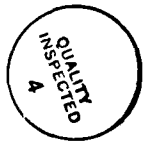

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Abstract

A constitutive equation is proposed with a view to describing the rate dependent mechanical response of metals at high temperatures. The equation is of the endochronic type and derives its physical foundations from deformation kinetics. Of importance is the fact that hardening is associated with a change in the energy barriers brought about by the inelastic deformation of a metal. The equation is used to describe the results, by Ohno and his associates, of experiments on the creep response of metals to piece-wise constant stress histories. The metal in the present case is 304 stainless steel at 600° C. It is shown that the theory gives analytical results that are in close agreement with experiment.

Of consequence is the fact that the constitutive equation applies to three-dimensional stress or strain histories and is thus not limited to those stress histories associated with creep.

1. Introduction

In the present paper we develop an endochronic theory of viscoplasticity which accounts for the history of strain and strain rate on the stress response of metals at high temperatures. The theory is based on the concepts of endochronic plasticity (see typically Ref.'s [1], [2], and [3]), however, the increment of intrinsic time scale is no longer proportional to the plastic strain path but depends also on the rate at which the path is traversed. The development of the theory is dealt with at length in the subsequent sections.

The resulting constitutive equation is used to analyze the creep response of 304 stainless steel to piece-wise constant shear stress histroies at 600°C and to compare the results with the experimentally determined creep response of the same material at this temperature as reported by Ohno et als. in Ref [4],

The constitutive behavior of metals at high temperature where the strain rate sensitivity of the mechanical response cannot be ignored, has been the subject of extensive research in recent years. We do not wish to give in this paper an exhaustive review of the literature on this subject but merely cite references which are typical of the enormous amount of work which is being done in this field. In this context, the works of Chaboche [5], Krieg [6], Malvern [7], Haisler [8], Bradley [9], Leckie [10], Krempl [11], Walker [12], Miller [13], and Hart [14], among others are mentioned. We also wish to cite the works of Ohashi et als. [15], and Murakami and Ohno [16], and Ohno et als. [4] who have been enjoying a measure of success in using a creep hardening surface theory in describing the creep response of metals to piece-wise constant stress histories, a subject with which we will be dealing in this paper. This aspect of the mechanical response of metals has given rise to greater difficulties than say, the stress response to piece-wise constant strain rate histories.

The work of Krausz [17] and his co-workers also occupies a significant position in the literature because of its more fundamental nature in that a microscopic theory of deformation kinetics is used to gain understanding of the mechanical response of metals at the bulk level.

In our initial approach to the subject, with specific attention to the strain (creep) response to piece-wise constant, pure shear stress histories, we used a strictly phenomenological approach, in the context of an endochronic theory. Specifically in one dimension we probed the data with the constitutive equation

$$e^p = \int_{0-}^{z} J(z-z') \frac{ds}{dz'} dz' \tag{1.1}$$

Henceforth convolution integrals such as the one appearing on the right hand side of eq. (1.1) will be given in symbolic form according to eq. (1.1a):

$$\int_{0-}^{z} J(z - z') \frac{ds}{dz'} dz' \overset{\text{def}}{=} J(z)*ds \tag{1.1a}$$

In equation (1.1) e^p has the same connotation as e^c, i.e., it is the inelastic shear strain where

$$\dot{e}^p = \dot{e}^c = \dot{e} - \frac{\dot{s}}{2\mu_0} \tag{1.2}$$

and μ_0 is the elastic shear modulus. We caution, however, that other definitions of e^c have been used in the literature. The intrinsic time z was defined by equation (1.3) where

$$dz = \frac{d\zeta}{f(\zeta,\dot{\zeta})} \tag{1.3}$$

and $\dot{\zeta} = |\dot{e}^c|$ is the usual fashion. The dependence of f on $\dot{\zeta}$ lends "strain rate sensitivity" to the equation, which otherwise would be strain rate insensitive.

While equations of the type (1.1), (1.2) and (1.3) were shown to give satisfactory results in the case of constant strian rate histories as demonstrated by Wu and Yip [18,19] and Lin and Wu [20,21], we found that these equations did not prove satisfactory in the case of piece-wise constant stress histories. With specific reference to the data of Ref. [4], it was found that f had to be essentially independent of z, to account for the periodic creep response to piece-wise constant cyclic histories. Thus, limiting f to a dependence on $\dot{\zeta}$ only resulted in a gross overestimate of the creep strain under cyclic conditions. Correcting f so as to match the data gave rise to oscillations in f which could not be accounted for by means of equation (1.3). Furthermore, the phenomenological approach did not give any hint as to the physical mechanism(s) responsible for such fluctuations in f. To overcome the difficulties we appealed to the theory of deformation kinetics in the context of the internal variable theory. The latter is treated briefly in Section 2 while the former is represented in detail in Section 3.

2. Internal Variable Theory

Irreversible thermodynamics of internal variables is now a well established field so we proceed to give a very breif outline of the theory for the sake of completeness. We limit ourselves to small strain fields. In the Helmholtz formulation the thermodynamic state is described by the free energy density ψ which is a function of the strain tensor $\underset{\sim}{\epsilon}$ the temperature T and n interval variables $\underset{\sim}{q}^r$ which, for thermomechanical processes, are tensors of the second order. The stress $\underset{\sim}{\sigma}$ and the entrpy density η are then given by equations (2.1) and (2.2)

$$\underset{\sim}{\sigma} = \frac{\partial \psi}{\partial \underset{\sim}{\epsilon}} \tag{2.1}$$

$$\eta = -\frac{\partial \psi}{\partial T} \tag{2.2}$$

In the case of a spatially uniform thermal field the condition of positive rate of irreversible entropy gives rise to the inequality

$$-\frac{\partial \psi}{\partial \underset{\sim}{q}^r} \cdot \dot{\underset{\sim}{q}}^r > 0 \ , \ \| \dot{\underset{\sim}{q}}^r \| \neq 0 \tag{2.3}$$

r, not summed.

In the linear version of the endochronic theory ψ is a quadratic function of its arguments and the evolution equations for $\underset{\sim}{q}^r$ have the form

$$\frac{\partial \psi}{\partial \underset{\sim}{q}^r} + \underset{\approx}{b}_r \cdot \frac{d\underset{\sim}{q}^r}{dz} = 0 \tag{2.4}$$

r not summed, where n such equations exist, one for each variable $\underset{\sim}{q}^r$, and $\underset{\approx}{b}_r$ are positive definite tensors of the fourth order.

When isothermal conditions prevail and the solid is initially isotropic and the tensors $\underset{\approx}{b}_r$ are constant, then equations (2.1) and (2.4) combive to give rise the integral constitutive equations

$$\underset{\sim}{s} = 2\mu(z)*d\underset{\sim}{e} \tag{2.5}$$

$$\sigma = 3K(z)*d\varepsilon \tag{2.6}$$

where $\underset{\sim}{s}$ is the stress deviator, $\underset{\sim}{e}$ the strain deviator and σ and ε the hydrostatic stress and strain respectively. The kernels $\mu(z)$ and $K(z)$ are sums of positive decaying exponential functions, i.e.,

$$\mu(z) = \sum_r \mu_r e^{-\alpha_r z}, \quad K(z) = \sum_r K_r e^{-\lambda_r z} \tag{2.7a,b}$$

where μ_r, α_r, K_r and λ_r are all non-negative.

In the generalized endochronic internal variable theory the evolution equations are expressed in terms of the intrinsic times of the mechanisms of internal motion. See Ref [22]. Specifically to each $\underset{\sim}{q}^r$ the theory ascribes an intrinsic time z_r such that the equations of evolution become

$$\frac{\partial \psi}{\partial \underset{\sim}{q}^r} + \underset{\approx}{b}_r \cdot \frac{d\underset{\sim}{q}^r}{dz_r} = 0 \text{ (r not summed)} \tag{2.8}$$

$r = 1, 2 \ldots n.$

In the endochronic theory of plasticity of metals as it has been used in the past

$$z_1 = z_2 = \ldots = z_n = z \tag{2.9}$$

and the elastic bulk modulus K in constant, i.e, the material is plastically incompressible. Also,

$$dz = \frac{d\zeta}{f(\zeta)}, \quad d\zeta = \| d\underset{\sim}{e}^p \| \tag{2.10a,b}$$

and

$$d\underset{\sim}{e}^p = d\underset{\sim}{e} - \frac{d\underset{\sim}{s}}{2\mu_0} \tag{2.11}$$

where μ_0 is the elastic shear modulus. The function f is positive and non decreasing. Thus in the case of plasticity, and in the context of the above assumptions, equations (2.5) and (2.6) have the form

$$\underset{\sim}{s} = 2\mu_0(z) * d\underset{\sim}{e} \tag{2.12}$$

$$G = 3K\epsilon \tag{2.13}$$

Substitution of equation (2.11) in equation (2.12) gives an equivalent constitutive equation which relates $\underset{\sim}{s}$ directly to the history of $\underset{\sim}{e}^p$. Thus

$$\underset{\sim}{s} = 2\rho(z) * d\underset{\sim}{e}^p \tag{2.14}$$

See Valanis, Ref. [23]. The relation between ρ and μ is given in terms of their Laplace transforms in equation (2.15).

$$\bar{\rho}\left(1 - \frac{p\bar{\mu}}{\mu_0}\right) = \bar{\mu} \tag{2.15}$$

It has been found that in the case of metals at room temperature ρ and f are well represented by the relations

$$\rho = \rho_0 \frac{1}{z^\alpha} e^{-kz}, \quad f = 1 - ae^{-bz} \tag{2.16a,b}$$

where ρ_0, α, a, b are positive and k is non-negative.

In the Gibbs formulation the thermodynamic state is described by the free energy density ϕ, which is a function of the stress tensor $\underset{\sim}{\sigma}$ the temperature T and n internal variables $\underset{\sim}{q}^r$ which, again, are second order tensors. The function ϕ is related to ψ by the equation

$$\phi = \psi - \sigma_{ij}\,\varepsilon_{ij} \tag{2.16}$$

The counterparts of equations (2.1) and (2.2) are

$$\underset{\sim}{\varepsilon} = -\frac{\partial \phi}{\partial \underset{\sim}{\sigma}} \tag{2.17}$$

and

$$\eta = -\frac{\partial \phi}{\partial T} \tag{2.18}$$

whereas the positive rate of irreversible entropy gives rise to the inequality

$$-\frac{\partial \phi}{\partial \underset{\sim}{q}^r} \cdot \dot{\underset{\sim}{q}}^r > 0, \quad \|\dot{q}^r\| \neq 0 \tag{2.19}$$

for each r, where $-\frac{\partial \phi}{\partial \underset{\sim}{q}r}$ is the internal microforce for the internal mechanism r while the evolution equations for $\underset{\sim}{q}^r$ are, in the case of the generalized endochronic formulation

$$\frac{\partial \phi}{\partial \underset{\sim}{q}^r} + \underset{\approx}{b}_r \cdot \frac{d\underset{\sim}{q}^r}{dz_r} = 0 \tag{2.20}$$

If we now stipulate that ϕ is quadratic in its variables and $dz_r = dz$ for all r, and $\underset{\approx}{b}_r$ are constant then in the case of isotropic materials and isothermal conditions, equations (2.17) and (2.20) combine to give the constitutive equations

$$\underset{\sim}{e} = \frac{1}{2} L(z) * d\underset{\sim}{s} \tag{2.21}$$

$$\varepsilon = \frac{1}{3} N(z) * d\sigma \tag{2.22}$$

When plastic incompressibility applies N is a constant and equal to $\frac{1}{K}$ (see equation (2.13). Also μ and L are related by equation (2.23).

$$\mu(z)*dL = H(z) \tag{2.23}$$

where H(z) is the unit step function.

In view of equation (2.11)

$$\underset{\sim}{e}^p = \underset{\sim}{e} - \frac{1}{2}\frac{\underset{\sim}{s}}{\mu_0} \tag{2.24}$$

Thus

$$\underset{\sim}{e}^p = \frac{1}{2} J(z)*d\underset{\sim}{s} \tag{2.25}$$

where

$$J(z) = -\frac{1}{2\mu_0} H(z) + L(z) \tag{2.26}$$

The functions J(z) and $\rho(z)$ are also related as shown in equation (2.27)

$$J(z)*d\rho = H(z) \tag{2.27}$$

Spectrum of Intrinsic Times

Of the n internal mechanisms let group r have an intrinsic time z_r. Then to the group r of internal variables n_r in number, there will correspond an intrinsic time z_r. Evidently $\sum_{r=1}^{m} n_r = n$ where m is the number of groups.

A straightforward analysis using equations (2.16) (2.17) and (2.20) when ϕ is a quadratic isotropic function of e_{ij} and $q_{ij}^{(r)}$ and $\underset{\sim}{b}_r$ are constant isotropic tensors leads to the equations

$$\underset{\sim}{e}^p = \sum_{r=1}^{n} J_r(z_r)*d\underset{\sim}{s} \tag{2.28}$$

$$\varepsilon^p = 0, \quad \varepsilon = \frac{1}{3} N\sigma \tag{2.29a,b}$$

where plastic incompressibility has been observed. They physical interpretation of equation (2.28) is that each of the m groups of mechanisms, say r, contributes to the total strain a partial strain $\underset{\sim}{e}^p_r$ such that

$$\sum_{r=1}^{m} \underset{\sim}{e}^p_r = \underset{\sim}{e}^p \tag{2.30}$$

where

$$\underset{\sim}{e}^p_r = J_r(z_r) * d\underset{\sim}{s} \tag{2.31}$$

The intrinsic time z_r is related to ζ by the equation

$$dz_r = \frac{d\zeta}{f_r}$$

where f_r is the hardening function of graph r.

The need for this more general approach which was presented in a previous reference [22], has been discussed in the introduction and has to do with the fact that one intrinsic time is just not sufficient to describe the creep response of metals to piece-wise constant stress histories. The physical justification for this possibility is discussed in the section on deformation kinetics, but simply put, it means that the hardening function f is not the same for all mechanisms at high temperatures, though the assumption of an f common to all $\underset{\sim}{q}^r$ suffices at room temperatures as demonstrated in our work on endochronic plasticity. The appeal to deformation kinetics is necessitated by the desire to determine how f is influenced by the micro-mechanical process which accompanies the inelastic deformation.

3. Deformation Kinetics

The theory of deformation kinetics was founded by Eyring [24] and its application to rate processes has been pursued by Eyring, Krausz [24] and their co-workers with a great deal of success.

In keeping with the ideas of deformation kinetics we attribute macromotion to additive effects of micromotions brought about by local distortion of atomic "energy barriers". At this point it is essential that we distinguish between diffusion of particles and diffusion of dislocations or vacancies. The distinction may be stated most simply in terms of the mean free path ℓ of a particle. In the case of particle diffusion ℓ is large compared to its counterpart in the case of dislocation or flaw diffusion. If a particle travels n units of distance "a" before coming to rest, then $\ell = na$. However, if a flaw travels n units of distance a, the mean free path of a particle is still a, because a different particle partakes each time in the motion of the flaw. Thus in the case of particle diffusion $n >> a$. Most important, however, is the fact that, in either case, each unit of motion consists of an atom moving across an energy barrier.

In addition we would expect that in the case of flaw or dislocation diffusion the energy barriers would be lower than those of particle diffusion. In fact, the activation energy of self diffusion is lower at low homologous temperatures (where dislocation motion is dominant) than at higher temperatures where particle diffusion dominates the process.

To apply the ideas of deformation kinetics to the viscoplastic deformation and flow of solids we appeal to a simple atomic model whereby prior to the application of stress each atom of the solid is situated at the bottom of a symmetric potential well. A typical potential well with the accompanying local potential surface is shown in Figure 1 by a solid line. The forward and backward barriers are equal and both have a height ϵ_0^r. When stress is applied,

the atoms will be displaced from their initial positions and the local potential surface of an atom will distort. The distorted potential surface is also shown. The effect of the distortion is to reduce the forward barrier by an amount w_f^r and increase the backward barrier by an amount w_b^r . It will be shown that this type of barrier distortion will give rise to an average forward motion of the atoms occupying potential wells with barriers ϵ_0^r.

Boltzmann Statistics. To determine quantitatively the effect of barrier distortion on the mean atomic motion we appeal to Boltzmann statistics. Accordingly the probability of finding an atom in an energy state ϵ_i is given by equation (3.1):

$$p_i = \alpha e^{-\beta\epsilon_i} \tag{3.1}$$

where

$$\alpha = 1/\sum_i e^{-\beta\epsilon_i} , \quad \beta = \frac{1}{kT} \tag{3.2a,b}$$

in the usual notation.

Let N_r be the number of atoms occupying potential wells with initial energy barriers ϵ_0^r . The probability that an atom is in an energy state greater than ϵ_0^r is p_0^r where

$$p_0^r = \sum_{\substack{r \\ \epsilon_i > \epsilon_0}}^{r} \alpha \exp(-\beta\epsilon_0^r) \tag{3.3}$$

States ϵ_i^r such that $\epsilon_i^r > \epsilon_0^r$ we have called activated states [25] differing from Eyring. Thus, the number of atoms in an activated state is $N_r p_0^r$. As the barriers are symmetric the probability that an atom will move forward is equal to the probability that it will move backwards so that the net motion (average displacement) of the atoms N_r is zero.

When the potential energy surface is distorted the number of atoms A_r partaking in a forward motion is now changed to

$$A_r = N_r \sum_{\substack{r \\ \varepsilon_i^r > \varepsilon_0^r - w_f^r}} \alpha \exp(-\beta \varepsilon_i^r) \tag{3.4}$$

while the number B_r of atoms partaking in as backward motion is

$$B_r = N_r \sum_{\substack{r \\ \varepsilon_i^r > \varepsilon_0^r + w_b^r}} \alpha \exp(-\beta \varepsilon_i^r) \tag{3.5}$$

The net number of atoms that partake in a forward motion is, thus, $A_r - B_r$ where

$$A_r - B_r = N_r \sum_{\substack{r \\ \varepsilon_b^r > \varepsilon_i^r > \varepsilon_0^r - w_f^r}} \alpha \exp(-\beta \varepsilon_i^r) \tag{3.6}$$

To evaluate the sum on the right had side of equation (3.6) we shall assume that w_f^r and w_b^r are both small. In this case we represent the distribution of energies ε_i^r in the vicinity of ε_0^r in terms of the local tangent to the distribution at ε_0^r by writing

$$\varepsilon_i^r = \varepsilon_0^r + k^r (i - i_0) \tag{3.7}$$

where i_0 is the value of i at $\varepsilon_i^r = \varepsilon_0^r$ and k^r is the slope of the distribution which is a function (in general) of ε_0^r. See Figure 2.

Substitution of the relation (3.7) in the sum on the right hand side of equation (3.6) leads to the simple expression

$$A_r - B_r = 2N_r \alpha \frac{e^{-\beta \varepsilon_0^r} \sin \beta_r w_r}{1 - e^{\beta k^r}} \tag{3.8}$$

where

$$w^r = \frac{w_f^r - w_b^r}{2} \quad , \quad \Delta\varepsilon_0^r = \frac{w_b + w_f^r}{2} \, . \tag{3.9,10}$$

We note that the terms $e^{-\beta\varepsilon_0^r}$ and $1 - e^{-\beta k^r}$ are related to the initial state, i.e., the barrier height and the energy distribution, while the terms $\sin \beta w_r$ and $\exp(-\beta\Delta\varepsilon_0^r)$ are brought about by barrier distortion.

The mean velocity v_r^p relative to the lattice of the atoms in group r may now be calculated in terms of the barrier distortion parameters N_r and $\Delta\varepsilon_0^r$. If λ is the mean lattice distance and τ_r is the average time taken by the atoms of group r to traverse that distance across the barrier ε_0^r then

$$v_r^p = (\lambda/\tau_r)(A_r - B_r)/N_r \tag{3.11}$$

We now define an internal variable q_r by the relation

$$\dot{q}_r = v_r^p/\lambda \tag{3.12}$$

Evidently in view of equation (3.11)

$$\dot{q}_r = (A_r - B_r)/N_r \tau_r \tag{3.13}$$

In view of equations (3.8) and (3.13)

$$\dot{q}_r = \frac{2\alpha e^{-\beta\varepsilon_0^r}}{1 - e^{-\beta k^r}} \left(\frac{1}{\tau_r}\right) e^{-\beta\Delta\varepsilon_0^r} \sinh \beta w_r \tag{3.14}$$

In so far as steady creep is concerned the assumption is usually made that of all the operating mechanisms only one survives in the steady state, i.e., $r = 1$ and w_1 is proportional to the stress (in one-dimensional stress fields). However, in the case of transient creep it is the local microforce, i.e., $-\frac{\partial\phi}{\partial q_r}$ on the group r that will determine the barrier distortion. Thus, following Ref. [25], we let w_r be proportional to $-\frac{\partial\phi}{\partial q_r}$ according to equation (3.15)

$$W_r = -C_r \frac{\partial \phi}{\partial q_r} \tag{3.15}$$

where C_r may depend on temperature.

Thus if we let

$$b_{00}^r = (1 - e^{-\beta k^r}) e^{\beta \varepsilon_0^r}/2\alpha \tag{3.16}$$

then in view of equations (3.14 - 3.16)

$$b_{00}^r \dot{q}_r + \frac{1}{\tau_r} e^{-\beta \Delta \varepsilon_0^r} \sinh (\beta C_r \frac{\partial \phi}{\partial q_r}) = 0 \tag{3.17}$$

<u>This is an "internal variable" form of the equation for the mean irreversible motion of the group r of paricles facing a potential barrier ε_0^r .</u>

<u>Discussion of equation (3.17)</u>. As we pointed out equation (3.17) establishes a physical meaning for the internal variables in that q_r is the mean displacement relative to the lattice of a group r of particles facing a potential barrier of magnitude ε_0^r . The above equation was published by Valanis and Lalwani in Ref. [25], with $\Delta \varepsilon_0^r = 0$. The appearance of the term $\Delta \varepsilon_0^r$ in equation (3.1) was inferred as a result of our effort to describe analytically the creep response of 306 stainless steel to piece-wise constant stress histories. This will be discussed in Section 4.

The time to traverse the barrier, i.e., τ_r is also of central importance in equation (3.17). Eyring used simplifying assumptions to arrive at the conclusion that τ_r is proportional to the square root of the ambient temperature. However, one can show that it depends at least in part on the barrier shape and height (Ref. [25]). In this work we have found that is also sensitive to the plastic strain rate. This is to be expected sind τ_r depends on the barrier conformation, which in turn depends on the plastic strain. The rate of plastic strain affects the rate of barrier distortion which must affect the traversal time τ_r.

Consider now two processes (a) and (b) the first of which is proceeding at a faster plastic strain rate than the second. With regard to the forward motion of a particle, the height of the barrier will be diminishing faster in case (a) than in (b), so that the forward moving particle will be encountering a consistently lower barrier in case (a) than in (b). It follows that the time to cross the barrier in case (a) will be shorter than in case (b). Thus

$$\tau_a < \tau_b \text{ whenever } \dot{\zeta}_a > \dot{\zeta}_b$$

The above inequality will be satisfied if

$$\tau = \frac{\tau_0}{g(\dot{\zeta})} \tag{3.18}$$

where τ_0 is a constant and g is a monotonically increasing function of $\dot{\zeta}$. In this work we have set

$$g(\dot{\zeta}) = \frac{1}{\dot{\zeta}^{1-m}} \tag{3.18a}$$

where m is a material constant.

Deformation kinetics is brought into accord with linear irreversible thermodynamics if in equation (3.17) the argument of the hyperbolic sine is sufficiently small for the approximation

$$\sinh\left(\beta C_r \frac{\partial \phi}{\partial q_r}\right) \sim \beta C_r \frac{\partial \phi}{\partial q_r} \tag{3.19}$$

to be appropriate. In this event equation (3.17) becomes a standard linear evolution equation, i.e.,

$$b^r \dot{q}_r + \frac{\partial \phi}{\partial q_r} = 0 \tag{3.20}$$

where

$$b^r = b_{00}^r \frac{\tau_r}{\beta C_r} e^{\beta \Delta \varepsilon_0^r} \tag{3.21}$$

In view of equations (3.18a), (3.20) and (3.21) one finds that the "endochronic" form of equation (3.20) is

$$b_0^r \frac{dq^r}{dz_r} + \frac{\partial \phi}{\partial q_r} = 0 \tag{3.22}$$

where

$$b_0^r = b_{00}^r \frac{\tau_0}{\beta C_r} \tag{3.23}$$

and

$$dz_r = \frac{dz}{\dot{\zeta}^m f_r} \tag{3.24}$$

$$f_r = e^{\beta \Delta \varepsilon_0^r} \tag{3.25}$$

Thus in deformation kinetics terms the rate sensitivity is attributable to the time to cross the barrier while the hardening (softening) is related to the change in the mean height of the barrier as a result of the stress history. Thus if $\Delta \varepsilon_0^r$ increases the material hardens while if it decreases the material softens in accord with our physical intuition regarding such processes.

4. Analysis of Piece-wise Constant Stress Histories in Pure Shear.

We begin with the integral

$$e^p = J(z)*ds \tag{4.1}$$

where e^p represents a shear creep strain component, s is the corresponding shear stress component and J the appropriate creep function. As usual

$$dz = \frac{d\zeta}{g(\dot{\zeta})f} \tag{4.2}$$

where g is the rate sensitivity function and f the hardening function. Also

$$d\zeta = k|de^p| \tag{4.3}$$

where k is an appropriate scalar constant. Typically, it e^p denotes a creep shear strain component and

$$d\zeta = \|d\underset{\sim}{e}^p\| \tag{4.4}$$

then $k = \sqrt{2}$ and J is the creep function in pure shear.

For our purposes it is more convenient to write equation (4.1) in the explicit form

$$e^p = \int_0^t J(z(t) - z(t')) \frac{ds}{dt'} dt' \tag{4.5}$$

for reasons that will become apparent.

4.1. Monotonic Creep in the Presence of a Constant Stress History

In this specific case

$$s(t) = s_0 H(t) \tag{4.6}$$

where H(t) is the unit step function whose "derivative" is the Dirac delta function. In this instance substitution of equation (4.6) in equation (4.5) gives the creep response in the simple form

$$e^p = s_0 J(z) \tag{4.7}$$

where s_0 is the amplitude of the step function of applied stress. It is apparent from equation (4.7) that if the form of J(z) is known then knowledge of z(t) determines creep strain in terms of the stress amplitude s_0. We caution that e^p is not necessarily linear in s since z(t) depends on s_0 as we shall demonstrate.

To this end differentiate equation (4.7) with respect to t and use equations (4.2) and (4.3) to find that under monotonic conditions

$$g(\dot{\zeta}) = k s_0 J'(z) \dot{z}^{-1} \tag{4.8}$$

where J'(z) is the derivative of J with respect to z. Thus

$$\dot{\zeta} = g^{-1} \{k s_0 J'(z) \dot{z}^{-1}\} \tag{4.9}$$

But from equations (4.7) and (4.3)

$$\dot{\zeta} = k s_0 J'(z) \dot{z} \tag{4.10}$$

Thus from equations (4.9) and (4.10)

$$\dot{z} = \frac{g^{-1}\{k s_0 J'(z) \dot{z}^{-1}\}}{k s_0 J'(z)} \tag{4.11}$$

Equation (4.11) gives z(t) by numerical integration if f is known.

To assign analytical forms to the functions J(z) and $g(\dot{\zeta})$ we appeal to experiment and the underlying assumptions of endochronic plasticity. It is common experience that metals become more strain rate sensitive as the temperature rises. However, the spirit of the endochronic theory is that this change is brought about not by a change in the form of J(z) but by virtue of $g(\dot{\zeta})$ which is evidently dependent on temperature even though this dependence is suppressed in equation (4.2).

At room temperature where rate effects are not significant J(z) is represented very closely by the analytical impression

$$J(z) = J_0 z^{\alpha} \tag{4.12}$$

where α is the vicinity of 0.85 for a number of metals. This form is retained by virtue of the above argument, at higher temperatures.

Experiments also indicate that under monotonic creep conditions

$$e^p = F(s_0)\, t^{\beta} \tag{4.13}$$

i.e., that the stress and time dependence of creep strain are factorable and that the time dependence is represented very closely by a power law. If during monotone creep f is a constant - which was found to be so for one component of the creep - then for equation (4.13) to hold $g(\dot{\zeta})$ must also be a power function. Thus we have set

$$g(\dot{\zeta}) = \dot{\zeta}^m \tag{4.14}$$

In view of these stipulations equation (4.11) now becomes

$$f^{\frac{1}{m}} \dot{z} = (k\, \alpha\, s_0\, J_0)^{\frac{1-m}{m}} z^{\frac{(\alpha-1)(1-m)}{m}} \tag{4.15}$$

Special solutions to equation (4.15). We proceed to give some special solutions to equation (4.15) when (a) f is constant and (b) when f is a power function of z. When f is constant the solution is given by equation (4.16):

$$f^{\frac{\beta'}{m}} z = A\, (k\, \alpha\, s_0\, J_0)^{n'} t^{\beta'} \tag{4.16}$$

where

$$A = \left(\frac{1}{\beta'}\right)^{\beta'}, \quad \beta' = m/1 + \alpha m - \alpha, \quad n' = \frac{1-m}{m}\beta' \tag{4.17}$$

In the case where $f = f_0 z^{\phi}$, where $\phi > 0$, the solution to equation (4.15) is given by equation (4.16) as before except that now the constant β' is given by equation (4.18):

$$\beta' = m/(1 + \alpha m - \alpha + \phi) \tag{4.18}$$

and the constant f on the left hand side of equation (4.16) is now denoted by f_0.

Knowing z(t) one may now calculate the creep strain e^p by use of equations (4.7) and (4.12), in conjunction with equation (4.16). Thus

$$e^p = A^{\alpha} f_0^{\frac{\beta}{m}} (k\alpha)^{n-1} (s_0 J_0)^n t^{\beta} \tag{4.19}$$

where

$$1 + \alpha n' = n, \quad \beta'\alpha = \beta \tag{4.20a,b}$$

Discussion. So far we have represented the creep strain by a single integral. We have also represented the creep function J(z), the strain rate sensitivity function $g(\dot{\zeta})$ and the hardening function $f(\zeta)$ by analytical forms of the power type. By analysis we then arrived at equation (4.19) which is basically of the form

$$e^p = B\, s_0^n\, t^{\beta} \tag{4.21}$$

where B is a constant, whereby the monotonic creep strain depends multiplicatively on the stress amplitude to the power n and the time to the power β. This form has appeared frequently in the literature where it has been arrived at by analysis of the data. It does not for all creep data and certainly not over the entire range of stress.

What is important, however, is that the creep strain depends on time according to a power law (equation (4.21) in accordance with observation as per equation (4.13) while the dependence on stress is of a more general type. One can change the dependence of e^p on s_0 by changing the analytical form of J(z) or $g(\dot{\zeta})$ or both but it seems that this would vitiate the dependence of e^p on a power function of t. Two other avenues are, however, available. One is to introduce a spectrum of intrinsic times, as discussed previously, i.e., a series

of integrals on the right hand side of equation (4.1). The other is to introduce a stress dependence in the hardening function f . This has been found to be the case in other materials such as polymers.

Specifically, if one sets

$$f = f_0(s_0)z^{\phi} \tag{4.22}$$

then under monotonic creep conditions in the presence of constant stress equation (4.19) will have the form given by equation (4.13), for an appropriate choice of the function f.

This approach alone, however, has been found inadequate to describe creep under cyclic piece-wise constant stress histories. This question as well as a constitutive equation involving more than one intrinsic time will be discussed in the next section.

5. Specific Constituitive Equations for 304 Stainless Steel

In the application of the theory to 304 stainless steel at 600°C and specifically to the experimental data generated by Ohno et als. (Ref. [4]), two terms were retained on the right hand side of equation (2.28), i.e.,

$$e^p = J_1(z_1)*ds + J_2(z_2)*ds \tag{5.1}$$

For the purposes of analysis and presentation of the results it is more convenient to write equation (5.1) in the form

$$e^p = e_1^p + e_2^p \tag{5.2}$$

where

$$e_r^p = J_r(z_r)*ds \tag{5.3}$$

r = 1, 2. In this case two hardening functions exist in the sense of equation (5.4)

$$dz_r = \frac{dz}{f_r} \cdot \frac{1}{\dot{\xi}m} \tag{5.4}$$

where m is a material constant found to be equal to 0.12. Also two creep functions $J_1(z)$ and $J_2(z)$ are needed and these were given the analytical forms shown in equation (5.5):

$$J_1 = J_1^0 z^{\alpha_1}, \quad J_2 = J_2^0 z^{\alpha_2} \tag{5.5 a,b}$$

where $\alpha = 0.836$, $\alpha_2 = 1$. $J_1^0 = 2.34 \times 10^{-3}$ MPa, $J_2^0 = 1.58 \times 10^{-3}$ MPa. It still remains to determine the form of the hardening functions. In order to match the monotonic data f_2 was represented by a power function of the form

$$f_2 = z^{\phi_2} \tag{5.6}$$

where $\phi_2 = 0.196$.

On the other hand f_1 could not be represented as a state function of, say, s and z, or any other variable for that matter. Rather, the experimental cyclic results of Ref. [4], gave strong indication that f_1 should be given in differential form of the type

$$d \log f_1 = dF(|s|, z_1)|_{z_1} \tag{5.7}$$

Note that the right hand side of equation (5.7) is not an exact differential and hence f is a function of the stress history. The physical implication of equation (5.7) is that a change in z_1 does not affect f_1 if during the change the absolute value of the stress s remains constant. A mathematically more explicit form for f_1 is

$$d \log f_1 = \frac{\partial F}{\partial |s|} (|s|, z_1)\, d|s| \tag{5.8}$$

The logarithmic form is not fortuitous but is a consequence of the physics of deformation kinetics and specifically equation (3.25) in view of which

$$\log f_1 = \beta \Delta \varepsilon_0' \tag{5.9}$$

$$d \log f_1 = \beta\, d(\Delta \varepsilon_0') \tag{5.10}$$

The implication is that in mechanism 1 the mean barrier height will change when the absolute value of stress changes but not otherwise. The constituitive description of the material is complete once the function $F(|s|, z_1)$ is known.

The function F is given below for various values of |s| (in MPa):

$$\begin{aligned} F(134.2, z_1) &= 2.25 + 1.3(1 - e^{-30z_1}) \\ F(120, z_1) &= F(134.2, z_1) \\ F(90, z_1) &= 2.19 + 0.9(1 - e^{-50z_1}) \\ F(60, z_1) &= 1.86 + 44(1 - e^{-85z_1}) \end{aligned} \tag{5.11a,b,c,d}$$

In addition the value of the hardening function at zero stress and zero value of z is set at 0.1054. This value together with the relation.

$$\log f(0,|s|_1) - \log f(0,|s|_2) = F(0,|s|_1 - F(0,|s|_2) \qquad (5.12)$$

where s_1 and s_2 are any two stress levels determines f for various values of the initial stress applied at the onset of a creep experiment.

6. Comparison with Piece-wise Constant Stress Experimetns of Ohno et als.

Monotonic Creep Experiments

The strain response to a constant stress history is obtained by application of equation (4.19) in conjunction with equation (5.2). Specifically

$$e^p = \sum_{r=1}^{2} A^{\alpha_r} f_{0r}^{-(\frac{\beta}{n})_r} (k\alpha_r)^{n_r-1} (s_0 J_{0r})^{n_r} t^{\beta_r} \tag{6.1}$$

in the presence of the constraint

$$\beta_1 = \beta_2 = \beta, \; n_1 = n_2 = n, \; m_1 = m_2 = m \tag{6.2}$$

so that equation (6.1) becomes

$$e^p = (s_0)^n t^{\beta} \sum_{r=1}^{2} A^{\alpha_r} (f_{0r})^{\frac{\beta}{m}} (k\alpha_r)^{n-1} J_{0r}^{n} \tag{6.3}$$

In the case of the linear model (2), f_{02} is a constant. However, in the non-linear model (1), f_{01} is a function of $|s|$. This dependence is determined by adjusting f_0, for various values of $|s|$, so as to obtain optimal agreement between theory and experiment in the case of monotonic creep. The function $f_{01}(|s|)$ is shown in Figure 3. With all the other constants known, the descriptive capability of the theory is shown in Figure 4.

Cyclic Creep Experiments

Following Section 2 let e_r^p be the r'th partial shear strain such that

$$e_r = J_r(z_r) * ds \tag{6.4}$$

and

$$e^p = \sum_{r=1}^{m} e_r^p \tag{6.5}$$

where in our case m = 2. For our purposes it is more convenient to write equation (6.4) in the form

$$e_r^p = \int_0^t J_r[z_r(t) - z(t^r)] \frac{ds}{dt^r} \, dt' \tag{6.6}$$

in the specific case of piece-wise constant stress histories of the type considered by Ohno et als.

$$\frac{ds}{dt} = s_0\{\delta(t) - a\,\delta(t-t_1) + a\,\delta(t-t_2) \ldots . \} \tag{6.7}$$

where $\delta(t)$ is the Dirac delta-function, s_0 is the initial stress amplitude and a is a constant. In this set of experiments two parameters s_0 and "a" define the history of stress - in addition to the reversal times t_1, t_2 ... t_n. Substitution of equation (6.7) in equation (6.6) and integration gives the explicit result

$$e_r^p = s_0 \{J_r(z_r) + a \sum_{n=1}^{N} (-1)^r J_r(z_r - z_{rn})\} \tag{6.8}$$

or

$$e_r^p = s_0\, y(z_r) \tag{6.9}$$

where $y(z_r)$ represents the bracket on the right hand side of equation (6.8). We differentiate equation (6.9) to obtain

$$\dot{e}_r^p = s_0\, \hat{y}(z_r)\, \dot{z}_r \tag{6.10}$$

where $\hat{y} = \frac{dy}{dz_r}$. Use of equation (5.4) then gives the result

$$\dot{e}_r^p = s_0\, \hat{y}\, \frac{\dot{\zeta}}{f_r z^m} \tag{6.11}$$

Now we take absolute values of both sides of equation (6.11) and use equation (4.3) to obtain

$$f_r \dot{\zeta}^m = s_0 \, k|\hat{y}| \tag{6.12}$$

But in view of equation (5.4)

$$\dot{\zeta} = (f_r \dot{z}_r)^{\frac{1}{1-m}} \tag{6.13}$$

Equations (6.12) and (6.13) combine to give the following differential relation between dz_r and dt

$$dt = \frac{f_r^{\frac{1}{m}} dz_r}{|k s_0 \hat{y}|^{\frac{1-m}{m}}} \tag{6.14}$$

Integration of equation (6.14) gives the relation between z_r and t. Substitution of $z_r(t)$ in equation (6.8) then gives the desired relation $e_r^p(t)$ and, therefore, $e^p(t)$ upon use of equation (6.5).

In our particular case

$$J_r = J_{0r} z_r^{\alpha_r} \tag{6.15}$$

and

$$y_r = \alpha_r J_{0r} z_r^{\alpha_1} + \sum_{n=1}^{N} (-1)^r J_{0r} (a_r - a_{rn})^{\alpha_r} \tag{6.16}$$

In Figures 5, 6 and 7 we show the experimental values of $e^p(t)$ obtained in Ref. [4] for the stress histories shown. Also shown are the analytical functions $e^p(t)$ obtained (a) by the use of the present theory and (b) as reported by Ohno et als. in Ref. [4] using their own theory.

The most significant difference in the predictive capability of the two theories lies in their depiction of the creep recovery slope at points of stress reversal. The endochronic theory predicts an infinite slope, in agreement with experiment, while the theory by Ohno et als. depicts a finite much shallower slope. Overall the predictive capability of the endochronic theory is very good for the type of stress histories discussed here.

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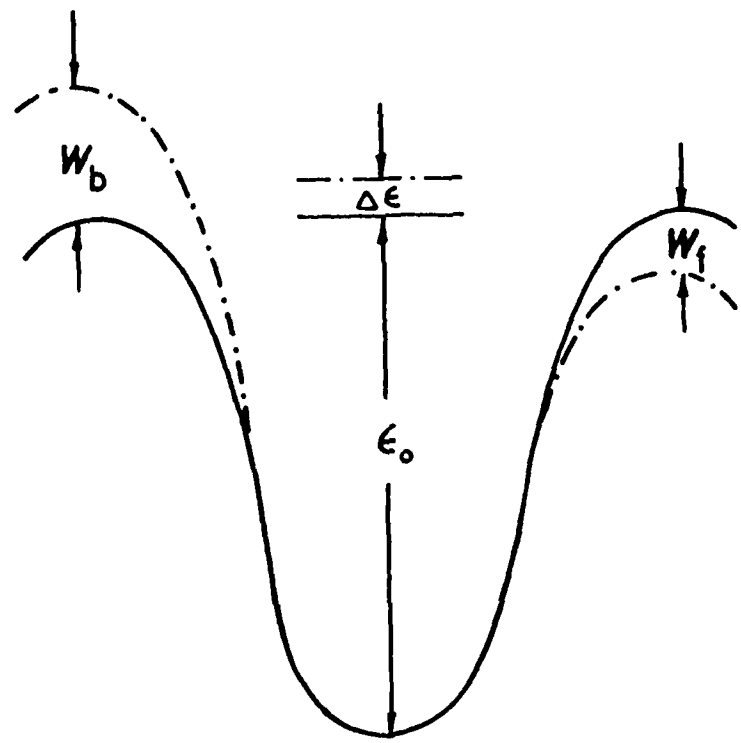

Fig. 1 Initial and Disturbed Barrier Surface

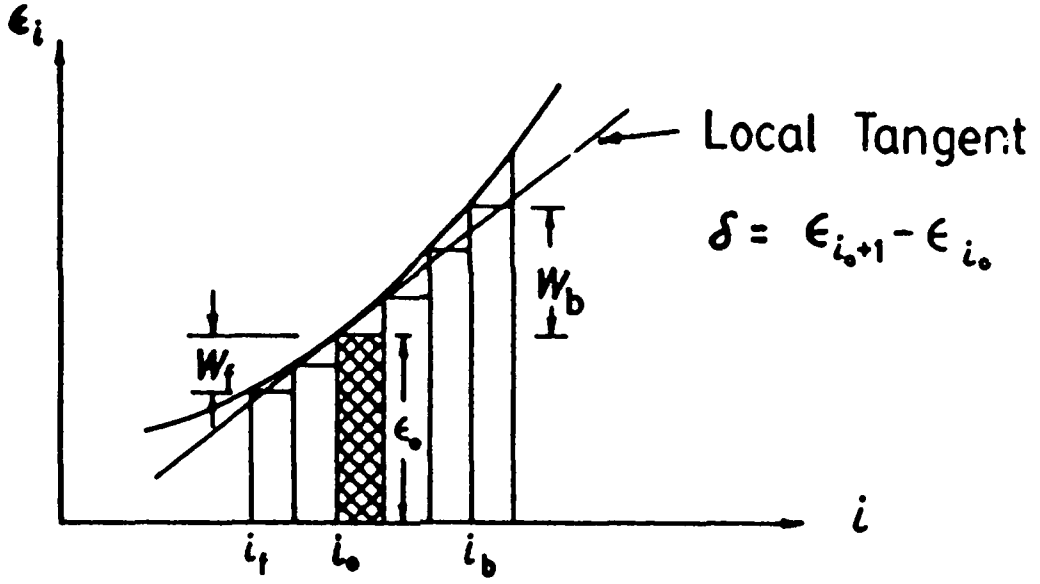

Fig. 2 Energy Distribution in Vicinity of ε_o

$$\epsilon_i = \epsilon_{i_o} + (i - i_o)\,\delta$$

$$W_b = (i_b - i_o)\,\delta$$

$$W_f = (i_o - i_f)\,\delta$$

$$\epsilon_o = \epsilon_{i_o}$$

$$P_f - P_b = \frac{2e^{-\beta\epsilon_o}}{1 - e^{-\beta\delta}}\, e^{-\beta\Delta\epsilon} \sinh \beta\bar{W}$$

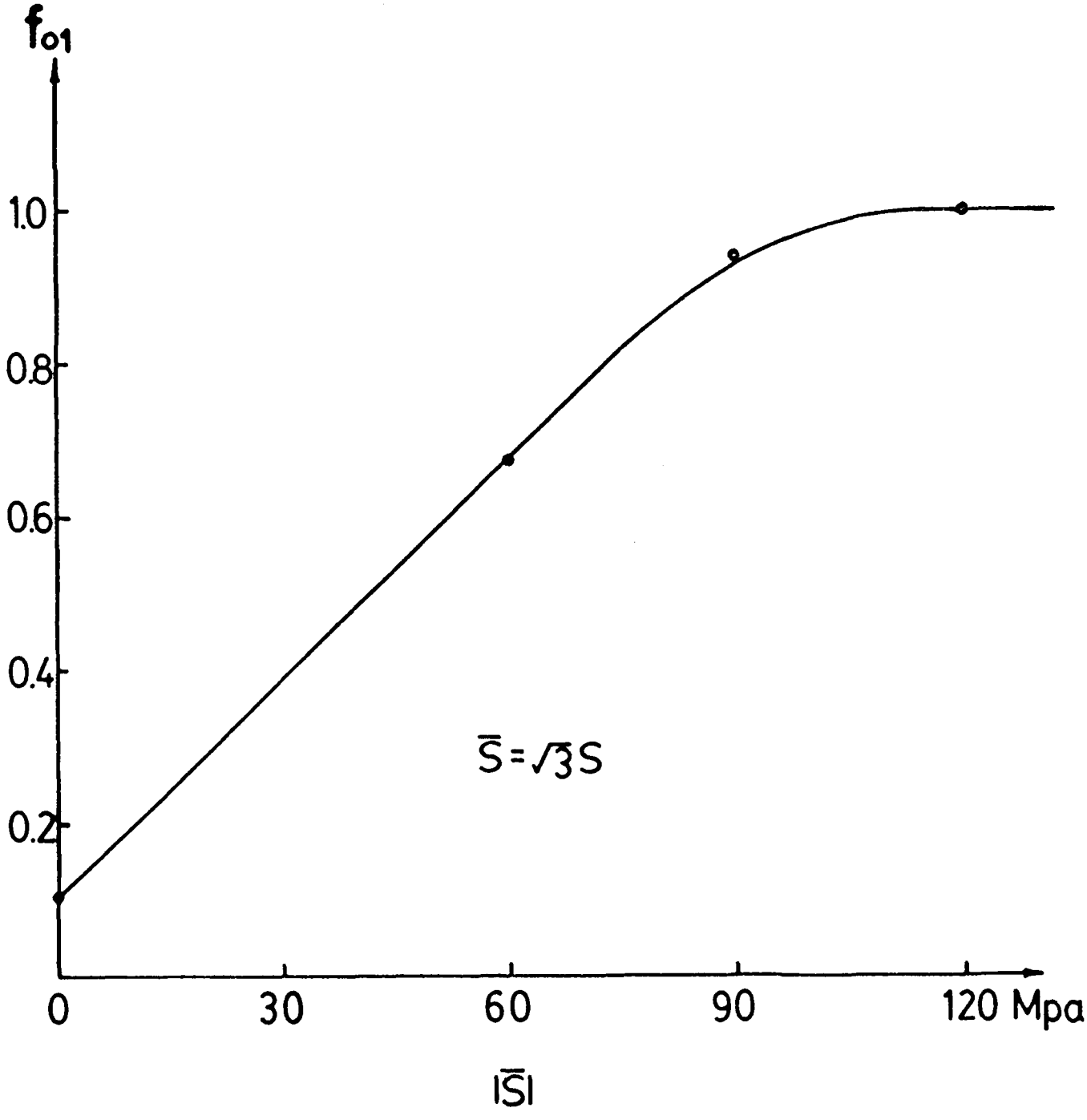

Fig. 3 Relation of $f_{o1}(|\bar{S}|)$

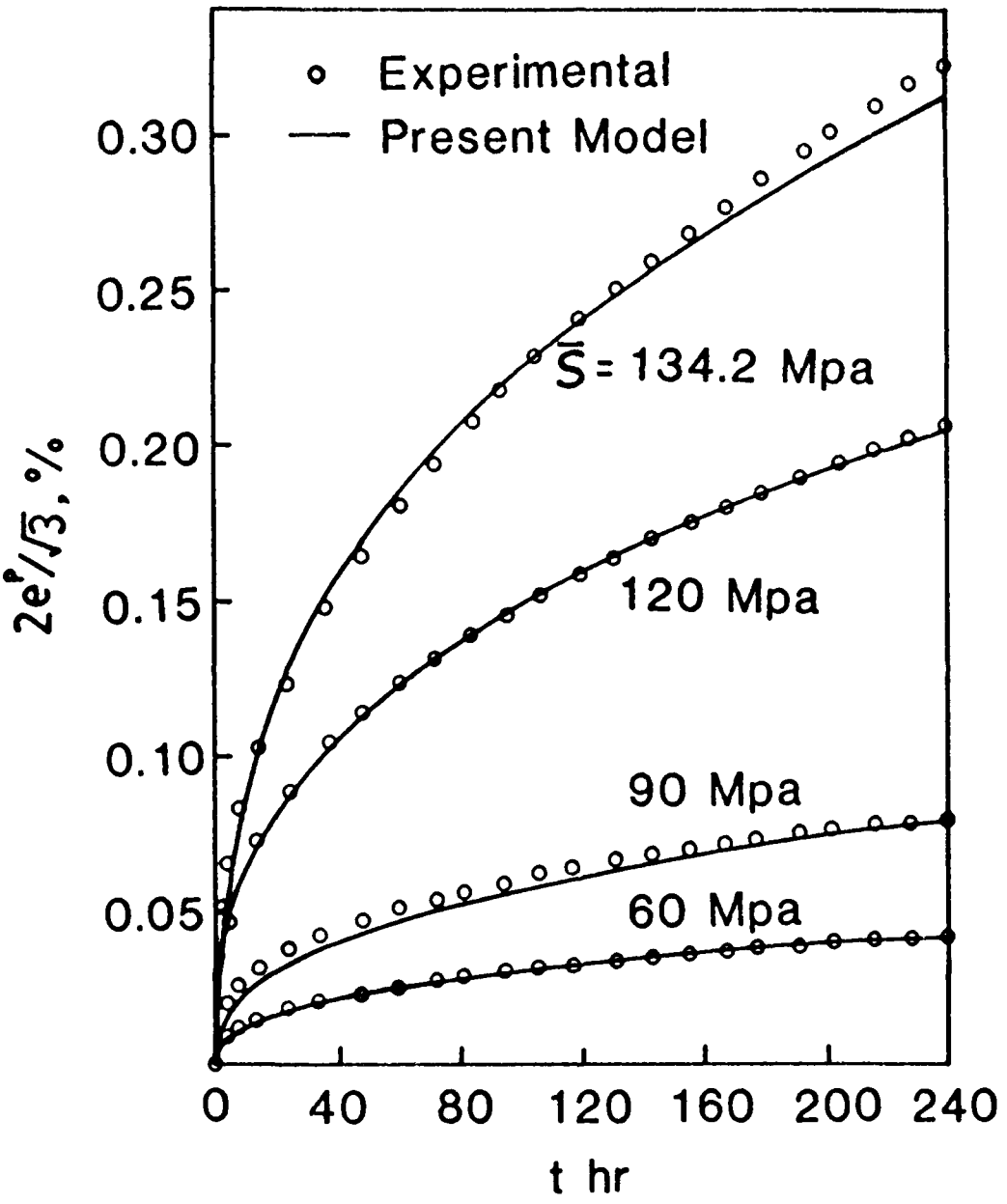

Fig. 4 Monotonic Creep Tests

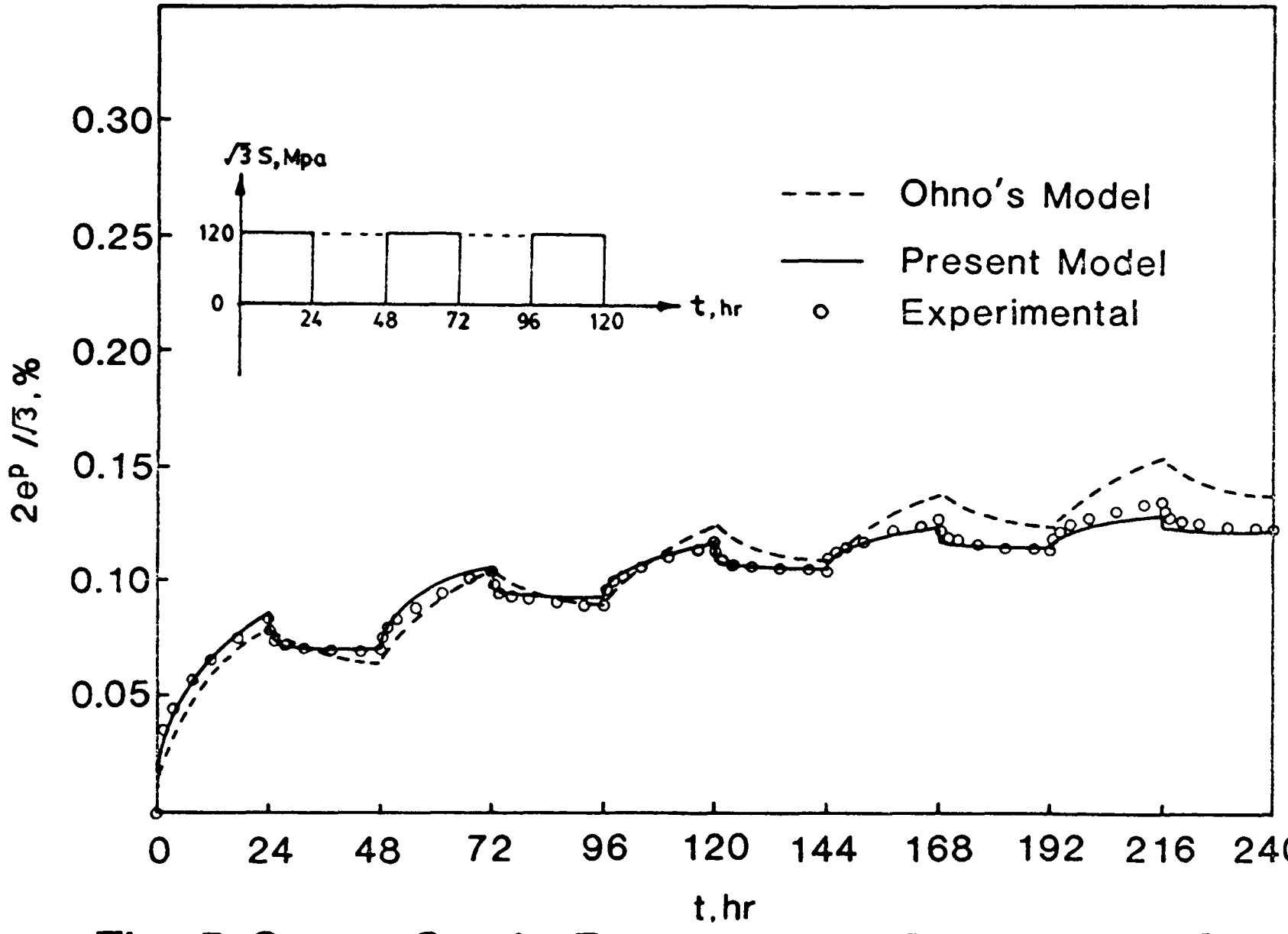

Fig. 5 Creep Strain Response to Piece-wise Constant Shear Stress History ($\sqrt{3}$ S = 120 Mpa, a = 1.0)

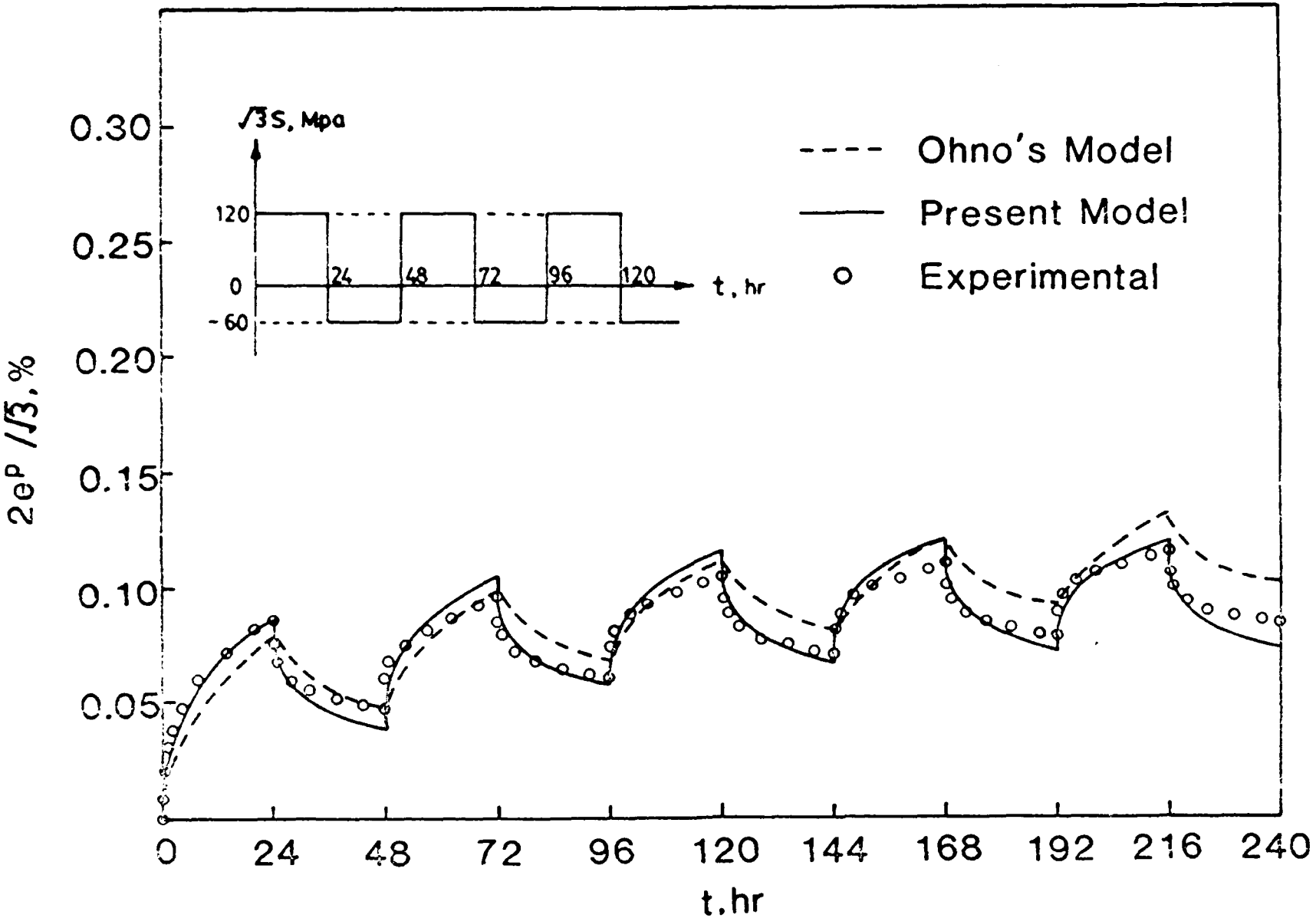

Fig. 6 Creep Strain Response to Piece-wise Constant Shear Stress History ($\sqrt{3}$ S = 120 Mpa, a = 1.5)

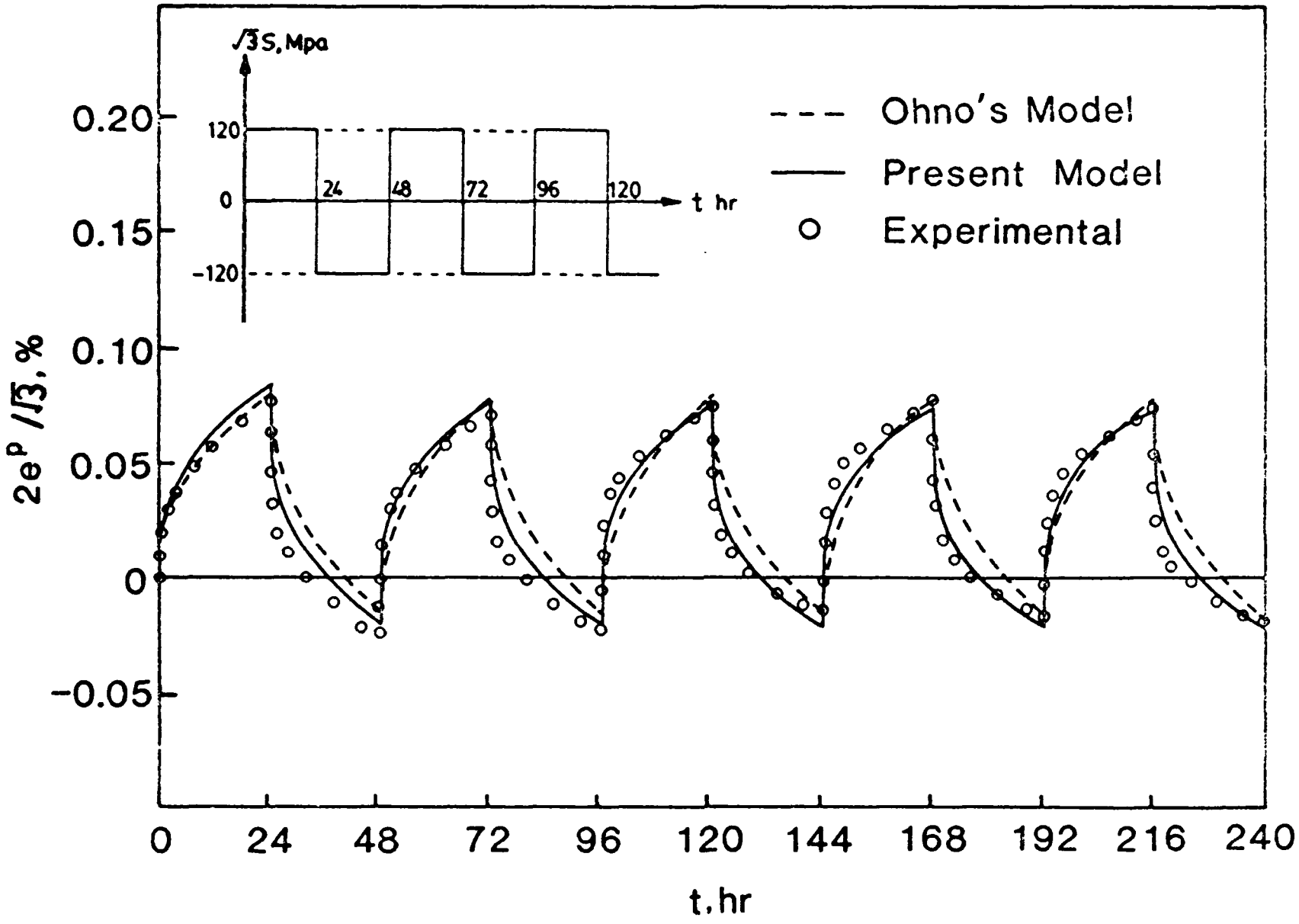

Fig.7 Creep Strain Response to Piece-wise Constant Shear Stress History ($\sqrt{3}S = 120$ Mpa, $a = 2.0$)

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A constitutive equation is proposed with a view to describing the rate dependent mechanical response of metals at high temperatures. The equation is of the endochronic type and derives its physical foundations from deformation kinetics. Of importance is the fact that hardening is associated with a change in the energy barriers brought about by the inelastic deformation of a metal. The equation is used to describe the results, by Ohno and his associates, of experiments on the creep response

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of metals to piece-wise constant stress histories. The metal in the present case is 304 stainless steel at 600° C. It is shown that the theory gives analytical results that are in close agreement with experiment.

Of consequence is the fact that the constitutive equation applies to three-dimensional stress or strain histories and is thus not limited to those stress histories associated with creep.

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